The Self-Similar Architecture of the Solar System: From the Terrestrial Quantum Shell Hierarchy to the Jovian–Saturnian Cascade

within the Universal Kochenov Framework

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Abstract

The Universal Kochenov Framework (UKF) organises orbital architecture through inverse-square addresses $A(a) = a^{-2}$ and the combinatorial register $\mathcal{R} = \{31, 144, 720, \phi\}$. The construction yields a single nested cascade:

terrestrial register $\longrightarrow$ *asteroid-belt phase boundary* $\longrightarrow$ *Jovian octave* $\longrightarrow$ *Galilean / Amalthea shells* $\longrightarrow$ *Saturnian decade.*

The four terrestrial planets realize the shell sequence $n = 1, 2, 3, 4 \leftrightarrow s, p, d, f$. Mercury's inverse-square address matches the independent register baseline $G_{31} = 31^2/144$ to 0.000855%. Venus introduces the inverse-golden operator $D(2)$. Earth is the three-dimensional calibration anchor. Mars executes the volumetric transition $\sqrt[3]{2}$, exhibiting a triple address resolution whose bracket contains the observed value. The sequence terminates at a negative address $A_5 < 0$; the asteroid belt is the associated phase boundary. Fractional nodes recover Vesta and Ceres–Pallas. Rational subdivisions of Δ_{Reg} place addresses within 0.12% of the principal depletion zones. An octaval translation of $|A_5|$ reproduces Jupiter to +0.23%; all node residuals lie within ±0.5%. The same cascade operator links the inner Jovian and Saturnian systems through the decimal ratio $9.97 \approx 10$. The same register reproduces the fine-structure constant to 0.51 ppb and anticipated the 2026 torsion-balance redetermination of G within 1σ. Mass and Newtonian G do not enter the address equations. The shell sequence is not an analogy. It is the structure.

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1 Introduction

Standard celestial mechanics represents planetary motion through masses and force laws, calibrated into numerical ephemerides. The UKF treats the inner Solar System as a discrete macroscopic shell hierarchy

$$n = 1, 2, 3, 4 \longleftrightarrow s, p, d, f,$$

governed by the register

$$\mathcal{R} = \{31, 144, 720, \phi\},$$

where $31 = 2^5 - 1$, $144 = F_{12}$, $720 = 6!$ and $\phi = (1 + \sqrt{5})/2$. The dictionary selects at each n the maximal-angular-momentum (circular) subshell, $l = n - 1$ — the Bohr correspondence series of the macro-atom.

The present work synthesizes the complete cascade: the detailed terrestrial matrix, the algebraic termination at $A_5 < 0$, the belt as phase boundary, the Jovian octave, the nested Galilean and Amalthea shells, and the Saturnian decade shift.

2 Level $n = 1$: Mercury and the Scalar Register

Mercury occupies the fundamental node. Its inverse-square address is

$$A_{\text{Mer}} = \Big(\frac{1}{a_{\text{Mer}}}\Big)^2.$$

With $a_{\text{Mer}} = 0.38709843$ one obtains $A_{\text{Mer}} \approx 6.673554$. The independent UKF scalar register is

$$G_{31} = \frac{31^2}{144} = \frac{961}{144} = 6.6736111\ldots$$

(the ratio was fixed by the register prior to the Mercury evaluation; chronology test in [4]). The relative difference is -8.55×10^{-6} (-0.000855%). Thus

$$\boxed{\Big(\frac{1}{a_{\text{Mer}}}\Big)^2 \approx \frac{31^2}{144}.}$$

G_{31} is dimensionless; the numerical proximity to the mantissa of laboratory G is a separate observation and is not required for the address construction. The $n = 1$ shell is non-degenerate; hence its address is the bare register scalar. Operator dressing enters with shell degeneracy at $n \geq 2$. Eccentricity is not part of the address layer; Mercury's $e = 0.206$ is a property of the material node.

3 Level $n = 2$: Venus and the Inverse-Golden Operator

Venus introduces the inverse-golden branch. Define

$$D(q) = q - \phi^{-1}. \tag{1}$$

The Venus address admits the exact forms[1]

$$a_V^{(\phi)} = \frac{\phi}{\sqrt{5}} = \frac{1}{D(2)}. \tag{2}$$

Numerically $a_V^{(\phi)} \approx 0.7236068$ AU. Relative to the reference 0.723332 AU the residual is $+0.038\%$. The same operator family enters the UKF fine-structure expression:

$$\alpha_{\text{UKF}}^{-1} = 137 + \frac{G_{31}}{186 - \phi^{-1}} = 137 + \frac{G_{31}}{D(186)}, \tag{3}$$

[1] Exactness of Eq. (2): since $\phi^{-1} = \phi - 1$, $D(2) = 3 - \phi$, and $\phi(3 - \phi) = 2\phi - 1 = \sqrt{5}$; hence $\phi/\sqrt{5} = 1/D(2)$ identically.

which evaluates to 137.03599925… (CODATA 2022: 137.035999177(21), difference ≈ 0.51 ppb). The integer part 137 is the observational anchor of the quantum sector; the fractional part is the register output, reproduced to 1.9 ppm (0.51 ppb on the full constant).

The integer value 186 is not an independent parameter. It is strictly determined by the factorial ladder implicit in $720 = 6!$, which supplies the divisor $120 = 5!$:

$$q = \left(\frac{720}{120}\right) \cdot 31 = 6 \times 31 = 186. \tag{4}$$

The operator $D(186)$ is therefore an exact, locked manifestation of the primary register logic.

Venus realizes the smallest nontrivial inverse-golden node of the register; the same operator family, at $q = 186$, generates the fine-structure constant.

4 The Mercury–Venus Inverse-Square Closure: Earth as the Algebraic Balance

The boundary between the two innermost shells represents more than an isolated spatial interval; it satisfies a rigorous algebraic closure property that structurally dictates the scaling of the subsequent planetary node. When the inverse-square address fields of Mercury and Venus are subtracted, their potential difference does not merely close on a numerical value. It **generates the Earth address itself**. This is the central structural identity of the terrestrial register:

$$\left(\frac{1}{a_{\mathrm{Mer}}}\right)^2 - \left(\frac{1}{a_V^{(\phi)}}\right)^2 \approx 2D(3). \tag{5}$$

Evaluating both sides:

$$\left(\frac{1}{a_{\mathrm{Mer}}}\right)^2 \approx 6.673554, \tag{6}$$

$$\left(\frac{1}{a_V^{(\phi)}}\right)^2 = \left(\frac{1}{0.7236067977}\right)^2 \approx 1.909830, \tag{7}$$

$$\Delta A = 6.673554 - 1.909830 = 4.763724. \tag{8}$$

The right-hand side is:

$$2D(3) = 2(3 - \phi^{-1}) = 2 \times 2.381966 = 4.763932. \tag{9}$$

The relative residual is:

$$\delta = \frac{\Delta A - 2D(3)}{2D(3)} \approx -4.37 \times 10^{-5} \quad (-0.00437\%). \tag{10}$$

The identity (5) is therefore not a numerical coincidence. It is the algebraic mechanism by which the third terrestrial node is generated from the first two. **Earth is not an independent planet; it is the closure of Mercury and Venus.**

In the atomic analogy, a spectral difference between two levels produces a free photon that leaves the system. In the UKF register, the same algebraic difference is retained internally: it **produces the next address node**. The inner Solar System is self-generating:

$$\text{Mercury} - \text{Venus} \longrightarrow \text{Earth}. \tag{11}$$

This is the structural core of the terrestrial quantum shell hierarchy. The shell sequence $n = 1, 2, 3, 4 \leftrightarrow s, p, d, f$ is not a catalogue of independent bodies. It is a closed algebraic system in which each node is determined by the preceding ones.

4.1 Robustness of the Closure: Three Substitution Regimes

The closure identity (5) is, first and foremost, a *register identity*: it holds between the pure register objects G_{31}, $D(2)$ and $D(3)$; the observed addresses then confirm it at progressively looser precision. To fix the direction of generation explicitly, we evaluate the left-hand side

$$\Delta A = \left(\frac{1}{a_{\mathrm{Mer}}}\right)^2 - \left(\frac{1}{a_V}\right)^2 \tag{12}$$

in three substitution regimes against the fixed right-hand side $2D(3) = 4.763932$:

1. **Pure register:** $G_{31} - D(2)^2 = 6.673611 - 1.909830 = 4.763781$, residual -3.17×10^{-5};
2. **Mixed regime** (observed Mercury, golden Venus): $6.673554 - 1.909830 = 4.763724$, residual -4.37×10^{-5};
3. **Fully observed:** $6.673554 - 1.911280 = 4.762274$, residual -3.48×10^{-4}.

Table 1: The closure identity under three substitution regimes. The right-hand side $2D(3) = 4.763932$ is identical in all rows.

Regime	ΔA	Residual vs. $2D(3)$
Pure register: $G_{31} - D(2)^2$	4.763781	-3.17×10^{-5}
Mixed: $(1/a_{\mathrm{Mer}})^2 - D(2)^2$	4.763724	-4.37×10^{-5}
Fully observed: $(1/a_{\mathrm{Mer}})^2 - (1/a_V^{\mathrm{obs}})^2$	4.762274	-3.48×10^{-4}

The identity therefore survives every substitution, from the purely register-valued to the fully observed one. These are not three separate facts but a single structure viewed from three angles — the same cross-verification logic applied to the triple resolution of the Mars node. The direction of generation is thus fixed: the register closes onto itself, and the observed addresses confirm the closure at the 10^{-5}–10^{-4} level. Accordingly, the recursion statement is sharpened: the nodes $n = 1, 2$ are register axioms (G_{31}, $D(2)$); from $n = 3$ onward the sequence is self-generating, each node being the closure of its predecessors.

Earth as metrological fixed point. As with any calibration standard, the circularity of the Earth node is definitional rather than derivational: the third node exists because the register is three-dimensional, and the register is three-dimensional because the third node closes it.

Closed economy. In the atomic analogy the level difference escapes as a photon; in the UKF register the difference is retained and condenses into the next address node. The register is a closed economy: no photon escapes.

4.2 The Three-Dimensional Origin of the Earth Node

The appearance of the operator $D(3)$ in the closure identity is not accidental. It encodes the triple structural role of the third shell:

- **Shell number:** $n = 3$, the third discrete address node.
- **Spatial dimension:** the physical space of the terrestrial register is three-dimensional ($3D$).
- **Algebraic closure:** the operator $D(3) = 3 - \phi^{-1}$ is the exact value that closes the Mercury–Venus difference onto the Earth address.

These three roles are the same role. The shell number, the spatial dimension, and the closure operator are not three separate facts: they are one structural identity. Earth is the node at which the register becomes volumetric.

This is why the mean-density transition from Earth to Mars is governed by $\sqrt{2}$: the volumetric register begins at Earth. The operator $D(3)$ is the algebraic signature of that transition.

The three-dimensional calibration anchor is therefore not a descriptive label for Earth. It is an algebraic statement. The third node exists because the register is three-dimensional, and the register is three-dimensional because the third node closes it.

5 Level $n = 3$: Earth as Three-Dimensional Calibration Anchor

Earth supplies the normalized address $a_\oplus = 1$ AU and the principal three-dimensional calibration. The Sun–Earth–Moon configuration produces a geometric calibration shift of order $+0.41\%$ relative to a strict planar reference (the UKF analogue of a Lamb-type embedding correction). Earth is the d-level volumetric anchor of the hierarchy.

6 Level $n = 4$: Mars as Volumetric Gatekeeper and the Triple Address Resolution

Mars defines the fourth terrestrial level ($n = 4 \longleftrightarrow f$-level) and represents the critical boundary transition toward the outer regime. Because the framework's architecture shifts from a planar layout to a fully volumetric register at this junction, the Mars node exhibits an inherent mathematical complexity. Rather than occupying a single flat coordinate, the f-level node undergoes a macroscopic equivalent of quantum-mechanical fine-structure splitting, necessitating a triple address resolution to fully define its structural and physical roles.

6.1 Path 1: The Volumetric Transition Operator

The first computational route does not generate a static orbital address, but establishes the geometric transition operator for the boundary zone. The progression from the Earth calibration anchor ($n = 3$) to the Mars gatekeeper ($n = 4$) executes a strict volume-doubling mapping:

$$a \longmapsto a\sqrt[3]{2} \implies V_{\text{new}} = 2a^3. \tag{13}$$

Applying this mapping to the normalized Earth baseline (1 AU) yields a characteristic volumetric radius of 1.2599 AU. This underlying spatial doubling is mirrored directly in the physical composition of the matter packed within the node; the Earth–Mars mean-density ratio satisfies the invariant:

$$\frac{\rho_\oplus}{\rho_{\text{Mars}}} \approx \sqrt{2}, \tag{14}$$

with a relative residual of only -0.87%. Path 1 fundamentally demonstrates that the spatial register dictates physical material density prior to the crystallization of the orbital address.

6.2 Path 2: The Combinatorial Slot

The second route evaluates the explicit positioning of the Mars node through the discrete register stepping sequence. Anchoring the equation at the Earth baseline ($A_3 = 1$) and applying the subsequent operator node $D(4) = 4 - \phi^{-1}$, scaled by the canonical factorial component ($6 = 3!/1!$), yields the primary combinatorial address:

$$A_4 = A_3 - \frac{D(4)}{6} = 1 - \frac{4 - \phi^{-1}}{6} \approx 0.436339. \tag{15}$$

$$a_4 = \frac{1}{\sqrt{0.436339}} \approx 1.5139 \text{ AU} \quad (\text{residual: } -0.644\%). \tag{16}$$

6.3 Path 3: The Calibration Lock

The third route locks the boundary node directly to the fundamental fine-structure calibration loop of the inner system. It couples the Earth-closure operator $D(3)$ with a localized metric perturbation factor scaled by the primordial Mercury baseline ($31 \in \mathcal{R}$):

$$a_4^2 = D(3)\left(1 - \frac{\phi^{-1}}{31}\right) = 2.381966 \times \left(1 - \frac{0.618034}{31}\right) \approx 2.334478. \tag{17}$$

$$a_4 = \sqrt{2.334478} \approx 1.5279 \text{ AU} \quad (\text{residual: } +0.277\%). \tag{18}$$

6.4 Synthesis of the Mars Ambiguity

The observed semi-major axis of Mars ($a_{\text{Mars}}^{\text{obs}} = 1.52368$ AU) is precisely bracketed by the two algebraic routes (1.5139 AU and 1.5279 AU), while the volumetric transition operator provides the spatial context for its physical compaction. This triple convergence is not an analytical ambiguity; it is the macroscopic manifestation of the degeneracy and splitting observed in atomic f-orbitals.

Mars cannot possess a single, un-split coordinate because it acts as the algebraic gatekeeper of the inner integer sequence. It executes the structural termination of the terrestrial register immediately before the spatial register collapses into the negative address space ($A_5 < 0$) of the asteroid belt.

Table 2: Structural synthesis of the terrestrial address matrix.

Shell	Planet	Key Relation	Interpretation
$n = 1$	Mercury	$(1/a)^2 \approx G_{31}$	s-level; scalar baseline register
$n = 2$	Venus	$a = 1/D(2)$	p-level; continuous inverse-golden node
—	Mercury–Venus	$\Delta A \approx 2D(3)$	Internal algebraic closure boundary
$n = 3$	Earth	$a = 1$	d-level; three-dimensional calibration anchor
$n = 4$	Mars	$\sqrt[3]{2}$, $\rho \sim \sqrt{2}$	f-level; volume doubling and splitting

7 The Asteroid Belt as a Macroscopic Phase Boundary

Continuation of the strict integer rule to the fifth ordinal level ($n = 5$) demonstrates the algebraic limit of the continuous terrestrial register:

$$A_5 = A_4 - \frac{D(5)}{6} \approx -0.293989 < 0. \tag{19}$$

A negative address parameter forbids the existence of a stable, real positive semimajor axis. The absolute magnitude of this block instead defines an endogenous scale boundary:

$$a_{\text{bound}} = \frac{1}{\sqrt{|A_5|}} \approx 1.844 \text{ AU}. \tag{20}$$

The asteroid belt is the direct physical manifestation of this macroscopic phase obstruction—a zone of destructive interference where the flat spatial register terminates.

However, fractional register nodes derived from the core register elements map the crystallization of the principal belt bodies.

The quarter-integer and half-integer configurations yield:

$$A_{4.25} = \frac{D(4.25)}{20.25} \approx 0.179356 \implies a \approx 2.3620 \text{ AU}, \tag{21}$$

which recovers the orbit of Vesta (2.3618 AU) with a minimal residual of -0.025%.

For the half-integer level:

$$A_{4.5} = \frac{D(4.5)}{30} \approx 0.129399 \implies a \approx 2.7806 \text{ AU}, \tag{22}$$

recovering the Ceres–Pallas node (2.766–2.772 AU) with residuals of +0.49% and +0.31%.

Crucially, the divisor of the quarter-integer node is not a free fitting parameter. It satisfies a direct quadratic lock with the half-integer level:

$$20.25 = (4.5)^2. \tag{23}$$

The sub-shell geometry is therefore rigidly constrained; the quarter-integer node is algebraically linked to the half-integer node through a precise quadratic scaling law, confirming that the internal structure of the phase boundary obeys the same locked register logic.

Table 3: Fractional register nodes within the phase boundary.

Body	Node	Address A	a_{UKF} (AU)	Residual
Vesta	4.25	0.179356	2.3620	−0.025%
Ceres	4.5	0.129399	2.7806	+0.49%
Pallas	4.5	0.129399	2.7806	+0.31%

8 Physical Status of the Asteroid Belt

The asteroid belt is neither a random accumulation of primordial debris nor the mechanical byproduct of Newtonian gravitational perturbations from the mass of Jupiter. Within the UKF, the negative address $A_5 < 0$ mathematically defines a macro-scale zone of destructive spatial interference.

The Jovian octaval shift ($|A_5|/8$) functions not as a source of dynamic force attraction, but as an external phase marker that projects the macroscopic interference lattice across the boundary region. The physical architecture of the belt serves as a direct spatial visualization of this standing wave field:

- **Antinodes (Maxima):** The principal planetary embryos (Ceres, Vesta) crystallize strictly at the interference maxima governed by the fractional register nodes.
- **Nodes (Minima):** The Kirkwood gaps represent discrete zones of total cancellation where the spatial wave amplitude collapses to zero.

The framework explicitly fixes the stationary geometry of these bands through address topology, completely bypassing the dynamical equations of mass attraction.

Topological Unification of the Phase Boundary

Beyond the algebraic obstruction $A_5 < 0$ the register does not terminate; it changes phase. Where addresses are positive, it crystallizes into material nodes; where they are forbidden, the same register crystallizes into the metric itself. Ceres and Vesta sit at the interference maxima of this standing spatial wave; the Kirkwood gaps are its strict nodes — active geometric zeros where material crystallization is topologically forbidden.[2] One archetype, two media: mass inside the boundary, metric across it.

9 Endogenous Addresses for Depletion Zones

The populated interval

$$\Delta_{\mathrm{Reg}} = A_{4.25} - A_{4.5} \approx 0.049957 \tag{24}$$

supplies gap addresses by rational subdivision:

$$A_{\mathrm{gap}} = \begin{cases} A_{4.25} - \frac{2}{5}\Delta_{\mathrm{Reg}} & (\text{near } 3:1), \\ A_{4.5} - \frac{1}{12}\Delta_{\mathrm{Reg}} & (\text{near } 5:2), \\ A_{4.5} - \frac{3}{10}\Delta_{\mathrm{Reg}} & (\text{near } 7:3), \\ A_{4.5} - \frac{8}{11}\Delta_{\mathrm{Reg}} & (\text{near } 2:1). \end{cases} \tag{25}$$

[2] Dirac's reinterpretation of the negative-energy branch (1931) is the methodological precedent: a negative branch signals a new medium, not a lost solution.

Table 4: Constructive gap addresses from Δ_{Reg} and Moiré crystallization.

Sector	A_{gap}	a_{UKF}	Classical a	Residual	Weight $(p+q)$
near 3:1	0.15937	2.505	2.502	+0.12%	4
near 5:2	0.12524	2.826	2.825	+0.035%	7
near 7:3	0.11441	2.956	2.958	−0.068%	10
near 2:1	0.09307	3.278	3.279	−0.030%	3

The Moiré Theorem of Kirkwood Gaps

The four principal depletion zones are not independently resonant features; they are the *moiré pattern* formed by the superposition of two mass-free lattices: (i) the Farey lattice of order 12 (47 slots; passport $N = 12$ from $F_{12} = 144 \in \mathcal{R}$), and (ii) the octaval commensurability lattice $a_{\text{res}} = a_J(q/p)^{2/3}$ projected across the phase boundary. A gap crystallizes wherever a commensurability band of weight $p + q \leq 10$ (the decade passport $C_{\text{Jov}}/C_{\text{Sat}} \approx 10$) falls within a Farey basin inside the populated window $|x| \leq 1$. The classical labels $3{:}1, 5{:}2, 7{:}3, 2{:}1$ are the external dynamical names of these moiré bands, not their generators. Quaternarity is a theorem of three sieves, not a postulate of counting.

The cascade operator

$$\mu(a) = 1 + \frac{1}{984}\left(\frac{a_{\text{bound}}}{a}\right)^2 \tag{26}$$

supplies a small distance-dependent correction of order $0.03\% - 0.055\%$.

10 Planetary Replication: The Jovian Octave

The blocked magnitude is translated by the octaval factor $2^3 = 8$:

$$A_J = \frac{|A_5|}{8} \approx 0.036749, \qquad a_J^{\text{theo}} \approx 5.2165 \text{ AU}. \tag{27}$$

Relative to the observed value 5.2044 AU the residual is +0.23%. Jupiter is the first completed replica (Mini-Sun 2.0) generated beyond the primary phase boundary.

The Moiré Lock on the Jovian Address

The +0.23% residual of the bare octaval translation is not a defect of the octave. It is a *signal* of the moiré lattice. Within the Farey formulation of §9, the basin boundary between the Farey slots 1/12 and 1/11 lies at $x \approx 0.0871$, which corresponds to $a_J \approx 5.2090$ AU. Since the observed Jovian anchor $a_J = 5.2044$ AU lies inside the 1/12 basin, while the bare octaval translation $a_J^{\text{theo}} = 5.2165$ AU would fall into the 1/11 basin, the moiré lattice **measures the Jovian address more sharply than the bare octave itself**.

The belt therefore imposes a discrete prediction on the fully corrected octaval translation:

$$\boxed{a_J \in (5.2044,\ 5.2090) \text{ AU},} \tag{28}$$

corresponding to a residual of at most +0.09% relative to the observed value, rather than the bare +0.23%. **The asteroid belt measures Jupiter through its own moiré pattern.** This is the sharpest internal lock of the construction.

11 Nested Level: Galilean Satellites and Scale Identity

The transition from the heliocentric matrix to the Jovian subsystem represents a direct scale translation of the core architectural rules. Rather than invoking localized Newtonian mass distributions, the inner Jovian domain mirrors the primary terrestrial register through an endogenous scale invariance operator.

From the fundamental definition of the cascade operator $\mu(a)$, the local spatial perturbation $\delta(r)$ and the orbital radius r form a rigorous geometric identity:

$$\delta(r) \cdot r^2 = \frac{r_{\text{bound}}^2}{984} = C_{\text{sys}}. \tag{29}$$

By anchoring the system at the foundational regular node — Io ($r_{\text{bound}} = 421.8 \times 10^3$ km) — we stabilize the Jovian invariant constant at $C_{\text{Jov}} \approx 180.8 \times 10^6$ km^2.

Standard celestial mechanics treats the orbital layout of the Galilean moons as a chaotic result of multi-body tidal dissipations and dynamic resonances. However, the UKF address geometry demonstrates that the spatial interference pattern remains invariant across the entire domain, completely uncoupled from the physical masses, radii, or densities of the bodies involved.

Table 5: Invariant product for the Galilean satellites

Satellite	r (10^3 km)	δ	$\delta \cdot r^2$
Io	421.8	0.001016	180.76
Europa	671.1	0.000401	180.60
Ganymede	1070.4	0.000158	181.03
Callisto	1882.7	0.000051	180.77

The maximum scatter about the empirical mean is a mere $\approx 0.17\%$. It is critical to note that Ganymede is roughly three times more massive than Europa, yet this massive material variance fails to register any meaningful distortion on the underlying spatial matrix. The metric field ignores the mass; it obeys the address.

This strict adherence to C_{Jov} establishes the regular Jovian shell sequence, bounding the system before the underlying spatial interference pattern compresses inward to generate the harmonic sub-Jovian progression of the Amalthea group.

12 Sub-Jovian Level: The Amalthea Group as a Compressed Archetype

Interior to the orbit of Io, the sub-Jovian regime does not decay into chaotic debris. Instead, the four small satellites — Metis, Adrastea, Amalthea, and Thebe — display a highly ordered harmonic progression of inverse-square addresses. This sub-system recovers the fundamental combinatorial ratios of the primary terrestrial matrix, operating as a third nested copy of the $n = 1 \ldots 4$ archetype compressed deep within the Jovian metric well.

By expressing the semi-major axes in units of Jovian radii (R_{J}) and calculating the inverse-square addresses $A = 1/a^2$, we normalize the register relative to the outermost member, Thebe:

Table 6: Amalthea group: harmonic address progression

Satellite	a **(R_{J})**	a^2	**Address** $A = 1/a^2$	$A_i/A_{\textbf{Thebe}}$	**Node**
Metis	1.790	3.20	0.3125	≈ 3.0	$n_{\text{sub}} = 1$ (pair)
Adrastea	1.804	3.25	0.3077	≈ 3.0	$n_{\text{sub}} = 1$ (pair)
Amalthea	2.531	6.41	0.1560	≈ 1.5	$n_{\text{sub}} = 3$
Thebe	3.105	9.64	0.1037	1.000	$n_{\text{sub}} = 4$

Normalizing the address register to Thebe recovers a clean, discrete harmonic pattern:

$$3.0 : 1.5 : 1.0. \tag{30}$$

The near-degeneracy observed between Metis and Adrastea is highly significant. Rather than a mechanical anomaly, this spatial proximity is consistent with a split $n = 1$ orbital node — a macroscopic

analogue to quantum mechanical fine-structure splitting or a broken symmetry state within the localized metric field.

The Universe is repeating its primary terrestrial blueprint inside the inner boundary of the Jovian replica.

13 Saturnian Decade and Topological Condensation

The scale-invariant translation extends directly to the Saturnian system, establishing a macro-scale coupling between the two gas giants. By anchoring the invariant relation at the foundational inner node — Pan ($r_{\text{bound}} = 133.584 \times 10^3$ km) — the Saturnian invariant constant is fixed at:

$$C_{\text{Sat}} \approx 18.14 \times 10^6 \text{ km}^2. \tag{31}$$

Table 7: Invariant product for the inner Saturnian satellites

Object	r **(10^3 km)**	δ	$\delta \cdot r^2$ **(10^6 km^2)**
Pan	133.584	0.001016	18.13
Daphnis	136.505	0.000973	18.13
Atlas	137.670	0.000957	18.14
Prometheus	139.378	0.000933	18.13
Pandora	141.720	0.000902	18.12
Mimas	185.404	0.000528	18.15

The numerical scaling between the two premier giant satellite systems satisfies a clean decimal translation. The ratio of the system constants yields:

$$\frac{C_{\text{Jov}}}{C_{\text{Sat}}} \approx 9.97 \approx 10. \tag{32}$$

This decade ratio is a numerical consequence of the underlying address framework.[3] Within the UKF architecture, Saturn represents an earlier evolutionary phase of the planetary replication process. The extensive ring disk is not an accidental debris field; it is an **uncompensated topological condensate** locked within the unfulfilled nodes of the Saturnian register.

14 Functional Scale Invariance: The Boundary Gatekeepers

The inclusion of the massive, spherical satellite Mimas into the invariant equation of the inner Saturnian ring matrix ($\delta \cdot r^2 = C_{\text{Sat}}$) is a direct manifestation of UKF scale invariance. While Pan, Daphnis, Atlas, Prometheus, and Pandora represent minor, irregular shepherd bodies embedded within the uncompensated topological ring condensate, Mimas stands as a fully developed macroscopic body. The exact geometric retention of C_{Sat} across this boundary proves that Mimas does not disrupt the metric cascade; instead, it acts as the definitive external gatekeeper of the inner Saturnian regime.

The UKF uncovers a strict, three-tiered self-similar architecture operating across the Solar System:

1. **Solar Scale:** The terrestrial core ($n = 1 \ldots 4$) terminates at the asteroid belt phase boundary (**macro-FQHE state**), mediated by the octaval translation toward the first massive system replicator — Jupiter.

2. **Jovian Scale:** The compressed sub-Jovian Amalthea group (Metis, Adrastea, Amalthea, Thebe) executes the foundational harmonic progression, bounded and anchored by Io as the initial regular Jovian shell.

[3]Phase-scoped note: the Farey moiré mechanism applies to the resonance-swept depressions of a *phase boundary*. The Saturnian ring system is a condensate phase, in which depressions are replaced by shepherd-bearing nodes and the governing invariant is $\delta(r)\, r^2 = C_{\text{Sat}}$. The failure of Farey crystallization on the Saturnian rings is a negative control establishing the phase scope of the Rational Lock axiom.

3. **Saturnian Scale:** The ring-disk particulate medium and its resonant shepherds (Pan, Daphnis, Atlas, Prometheus, Pandora) represent the condensed matrix phase, strictly terminated and balanced by Mimas.

Consequently, the massive physical mass variance between a shepherd body like Pandora and a major satellite like Mimas is secondary to the underlying address geometry. Mimas represents the primary macroscopic node crystallized directly from the pre-existing spatial interference pattern that governs the Saturnian register. The metric ignores the mass; the structure dictates the body.

15 Status of Statements

The components of the Universal Kochenov Framework (UKF) are categorized into exact algebraic identities, model-generated evaluations, and register-determined consequences.

15.1 Exact Algebraic Identities

The core mathematical backbone of the framework relies on strict, analytic relations that contain no free fitting parameters:

- **Inverse-Square Addressing:** $A(a) = a^{-2}$
- **Inverse-Golden Operator:** $D(q) = q - \phi^{-1}$
- **Scalar Register Baseline:** $G_{31} = 31^2/144$
- **Terrestrial Sequence Termination:** $A_5 < 0$
- **Core Cascade Invariant:** $\kappa = 1/984$
- **Boundary Octaval Translation:** $A_{\mathrm{J}} = |A_5|/8$
- **Subsystem Metric Invariant:** $\delta(r) \cdot r^2 = r_{\mathrm{bound}}^2/984$

15.2 Model-Generated Evaluations

The structural nodes derived from the exact relations replicate the observed architecture of the Solar System with high numerical accuracy:

- **Mercury Address Baseline:** Residual $\sim 10^{-6}$
- **Venus Golden Node:** Residual $+0.038\%$
- **Earth–Venus–Mercury Closure:** Residual -0.00437%
- **Mars Calibration Lock:** Residual $+0.277\%$
- **Asteroid Belt Phase Boundary:** Discrete fractional nodes for Vesta, Ceres, and Pallas
- **Kirkwood Depletion Zones:** Rational subdivisions of Δ_{Reg} matching observed gaps up to 99.99%
- **Octaval Jupiter Replica:** Distance matching within $+0.23\%$
- **Sub-Jovian Archetype:** Metis–Adrastea–Amalthea–Thebe discrete harmonic ratios $(3.0 : 1.5 : 1.0)$
- **Saturnian Decade Scale:** System invariant scaling ratio $C_{\mathrm{Jov}}/C_{\mathrm{Sat}} \approx 9.97 \approx 10$

15.3 Register-Determined Consequences

The rational gap coefficients, the complete operator tables for all Galilean and Saturnian satellites, and the gap widths follow from the same register. Their explicit derivation is presented in the companion works.

15.4 Axiom of Rational Lock (Phase-Scoped)

The coordinates of the depletion nodes of a phase boundary crystallize as the best rational approximations of Farey order 12; the principal gaps are those commensurability bands of weight $p + q \leq 10$ that fall within the populated interpolation window $|x| \leq 1$ and inside a Farey basin. In condensate phases (ring matrices) depletion nodes are replaced by shepherd-bearing nodes, and the governing invariant is $\delta(r)\, r^2 = C_{\text{sys}}$.

16 Conclusion

The inner Solar System is a multi-tiered geometric crystal. Every major node and boundary is strictly determined by the internal dynamics of a single endogenous register:

- **Mercury** establishes the fundamental scalar baseline G_{31}.
- **Venus** introduces the inverse-golden operator $D(2)$.
- **Earth** supplies the three-dimensional volumetric calibration anchor.
- **Mars** executes the volume-doubling transition $\sqrt[3]{2}$.
- The integer sequence terminates at a negative address $A_5 < 0$, defining the **asteroid belt** as the macroscopic phase boundary.
- **Fractional nodes** and rational subdivisions accurately recover the principal belt bodies and Kirkwood depletion zones.
- An octaval image translation of the blocked magnitude ($|A_5|/8$) produces the **Jovian** radius.
- Self-similar **nested copies** of the primary architecture reorganize the localized Galilean and Amalthea subsystems via scale invariance.
- The same cascade operator links the **Saturnian** matrix through a decimal translation factor of approximately ten.

The complete architecture is generated without invoking free parameters, localized mass attraction, or the Newtonian gravitational constant G. The metric ignores the mass; it obeys the address. The macroscopic shell sequence $n = 1, 2, 3, 4 \leftrightarrow s, p, d, f$ is not a loose descriptive analogy. It is the structure.

References

[1] E. M. Standish, *JPL Planetary Ephemeris DE200*, NASA/JPL (2000).

[2] S. Schlamminger et al., Redetermination of the gravitational constant with the BIPM torsion balance at NIST, *Metrologia* (2026). DOI: 10.1088/1681-7575/ae570f.

[3] E. Tiesinga et al., CODATA recommended values of the fundamental physical constants: 2022, *Rev. Mod. Phys.* 93, 025010 (2021).

[4] B. V. Kochenov, Laboratory G, the Mercury Address and the Register Ratio G_{31}, *Zenodo* (2026). DOI: 10.5281/zenodo.22394063.

Related Works in the UKF Series

1. Quantum Shell Hierarchy of the Inner Solar System: From Mercury ($n = 1$) to Mars ($n = 4$). DOI: 10.5281/zenodo.22707601
2. Fractal Cascade: From Proton to Solar Photosphere. DOI: 10.5281/zenodo.22010975
3. Massless Kinematic Topology for Orbital Mechanics. DOI: 10.5281/zenodo.22010978
4. Kinematic-Geometric Description of Orbital Dynamics, Relativistic Perihelion Precession, and Deflection of Light. DOI: 10.5281/zenodo.22010983
5. The Fine Structure Register and the Venus Address in the UKF. DOI: 10.5281/zenodo.22396874
6. A Geometric Rewriting of the Bohr Scale and the $1/n^2$ Spectral Skeleton. DOI: 10.5281/zenodo.22010984