Geometric G: From Earth's Geometry to the Gravitational Constant

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Abstract

We present an empirical geometric observation: when the gravitational constant G is expressed in units derived directly from the Earth (meter from the meridian quadrant, kilogram from one liter of water, second from the mean solar day), the length and mass dimensions reduce to a factor of 10^3 (to 0.003%, limited by the density of water $999.97\,\mathrm{kg/m^3}$), leaving a pure inverse square of time:

$$G_{\mathrm{Earth}} = \frac{31^2 \times \sqrt{5}}{\phi^{12}} \times 10^{-8}\,\mathrm{s}^{-2} \approx 6.67354674496\ldots \times 10^{-8}\,\mathrm{s}^{-2}.$$

In SI units this corresponds to a value lying between the two most precise laboratory measurements. No experimental G values were used in constructing the expression; experimental values are shown only for comparison.

1 Earth-Based Unit Definitions

Let M denote the length of one quarter of the Earth's meridian.

- Meter: $1\,\mathrm{m} = M/10^7$
- Decimeter: $1\,\mathrm{dm} = M/10^8$
- Kilogram: mass of $1\,\mathrm{dm}^3$ of water
- Second: $1\,\mathrm{s} = 1/86400$ of the mean solar day

2 Dimensional Reduction in Earth-Based Units

From these definitions:

$$1\,\mathrm{m}^3 = \frac{M^3}{10^{21}}, \qquad 1\,\mathrm{kg} \approx \frac{M^3}{10^{24}} \quad (\text{water density } 999.97\,\mathrm{kg/m^3}).$$

Hence

$$\frac{1\,\mathrm{m}^3}{1\,\mathrm{kg}} \approx \frac{M^3 \cdot 10^{-21}}{M^3 \cdot 10^{-24}} = 10^3 \quad (\text{to } 0.003\%).$$

Applying this to the Kochenov geometric formula

$$G_{\mathrm{Koch}} = \frac{31^2\sqrt{5}}{\phi^{12}} \times 10^{-11}\,\mathrm{m^3 kg^{-1} s^{-2}},$$

we obtain

$$G_{\mathrm{Earth}} = G_{\mathrm{Koch}} \times 10^3 = \frac{31^2\sqrt{5}}{\phi^{12}} \times 10^{-8}\,\mathrm{s^{-2}}.$$

All length and mass dependence cancels, yielding an effective s^{-2} dimensional form in this unit system.

3 Link Between the Second and the Golden Ratio

The number of seconds per mean solar day factorises as

$$86400 = 120 \times 720 = 120 \times 6!.$$

Independently, the golden ratio satisfies the Kochenov identity (derived in Appendix A)

$$\frac{\phi^{12}}{\sqrt{5}} = 144 + \frac{1}{720} + \varepsilon, \qquad \varepsilon \approx -1.39 \times 10^{-8}.$$

The same integer 720 appears both in the Earth's daily rotation and in the geometric expansion of $\phi^{12}/\sqrt{5}$. This reveals an unexpected numerical correspondence between Earth's rotation period and the golden ratio identity.

4 Comparison with Laboratory Measurements

Table 1: Comparison of G values

Source	Value ($\times 10^{-11}$)	Gap from G_{Earth}
Schlamminger (2014) [1]	6.6729 ± 0.0005	$+0.010\%$
G_{Earth} (this work)	$6.67354674496\ldots$	—
CODATA 2018 [2]	6.67430 ± 0.00015	-0.011%

The mutual disagreement between laboratories (0.021%) is larger than the deviation of our geometric value from either measurement. No experimental G values were used in the derivation; the numbers 31, 10^{11}, and $1/720$ originate from independent geometric constraints documented in [3, 4, 5, 6].

5 What This Is and Is Not

This work is:

- an empirical geometric parametrisation of G,
- a demonstration that G can be numerically represented using ϕ, 31, and $\sqrt{5}$,
- a dimensional reduction to s^{-2} in Earth-based units.

This work is not:

- a derivation of gravity from first principles,
- a modification of Newtonian or Einsteinian gravity,
- a claim that spacetime is discrete or spiral-structured,
- an attempt to explain the physical origin of ϕ, 31, or 720 beyond the references provided.

6 Limitations

Since the historical SI precursors (meter, kilogram) were defined using Earth-based quantities and the mass of water, the dimensional cancellation exhibited in Section 2 may partly reflect anthropogenic unit construction rather than fundamental physics. The numerical coincidences involving 720 and the second are presented as empirical observations, not as evidence of causal physical mechanisms.

7 Conclusion

The gravitational constant can be represented in a compact geometric form involving only the golden ratio, the integer 31, and $\sqrt{5}$. When expressed in units derived from Earth's meridian and the mass of water, G reduces to a pure inverse square of time. The same integer 720 that defines the second also appears in the geometric residual of $\phi^{12}/\sqrt{5}$. These observations may indicate a numerical connection between Earth's geometry, the golden ratio, and the measured value of G. The physical significance of this formulation remains an open question.

A Derivation of the Key Identity via Binet's Formula

The Fibonacci numbers satisfy Binet's formula:

$$F_n = \frac{\phi^n - \psi^n}{\sqrt{5}}, \qquad \psi = -\phi^{-1}.$$

For $n = 12$:

$$F_{12} = 144 = \frac{\phi^{12} - \phi^{-12}}{\sqrt{5}}.$$

Rearranging gives:

$$\frac{\phi^{12}}{\sqrt{5}} = 144 + \frac{\phi^{-12}}{\sqrt{5}}.$$

Now compute the residual ε:

$$\frac{\phi^{-12}}{\sqrt{5}} = \varepsilon + \frac{1}{720}, \qquad \varepsilon = \frac{\phi^{-12}}{\sqrt{5}} - \frac{1}{720}.$$

Numerically, $\phi^{-12} \approx 0.003105624$, so $\phi^{-12}/\sqrt{5} \approx 0.001388875\ldots$, while $1/720 = 0.001388888\ldots$. Hence

$$\varepsilon \approx -1.39 \times 10^{-8}.$$

Thus,

$$\frac{\phi^{12}}{\sqrt{5}} = 144 + \frac{1}{720} + \varepsilon, \qquad |\varepsilon| \approx 1.39 \times 10^{-8}.$$

References

[1] Schlamminger, S. et al. (2014). Measurement of Newton's gravitational constant. Phys. Rev. Lett.

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