Fractal Cascade: From Proton to Solar Photosphere

Universal Kochenov Framework (UKF)

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1. Cascade

$$\boxed{\lambda_p \xrightarrow{\times\Lambda} R_{\text{core}} \equiv a_{\text{Io}} \xrightarrow{\times\alpha^{-1}} a_{\text{Mercury}} \xrightarrow{\times S} \bar{\lambda}_e}$$

$$\Lambda = \frac{2 \times 144^3 \times 31^2}{720 \times 2.5} \times 10^{17} = 3.188367 \times 10^{23}$$

$$S = \frac{720 \times 2.5}{2 \times 144^3 \times 31^2} \times 10^{14} = 31\,364 \text{ km}$$

S — Space Step, the lattice spacing

$$\Lambda_e = \frac{a_{\text{Mercury}}}{\bar{\lambda}_e} = 1.49962 \times 10^{23}$$

2. Operator Register

$$\mathcal{M} = \{31,\ 144,\ 720,\ \phi,\ \sqrt{5},\ \alpha^{-1},\ \chi\} \quad (F_{12} = 144 \in \text{Fibonacci})$$

$$\chi_{\text{base}} = \frac{720}{2 \cdot 144} = 2.5, \qquad \chi \equiv \frac{m_n - m_p}{m_e} \approx 2.531$$

$$\frac{M_5}{M_7 - M_5} = \frac{31}{96}, \quad \frac{M_5^2}{M_7 - M_5} = \frac{961}{96} = 10.010416$$

$$\frac{M_5^2}{F_{12}} = \frac{31^2}{144} = \frac{961}{144} = 6.673611 \quad (\delta = -0.01\%)$$

$$\alpha^{-1} = 137.036, \qquad M_5 = 31, \qquad M_7 = 127$$

3. Proton → Io

$$\lambda_p = 1.321409855 \times 10^{-15} \text{ m}$$

$$\Lambda_{\text{UKF}} = 3.188367 \times 10^{23}$$

$$R_{\text{core}} \equiv a_{\text{Io}} = \lambda_p \times \Lambda_{\text{UKF}} = 4.21314 \times 10^8 \text{ m} = 421\,314 \text{ km}$$

Verification:

JPL/NASA mean semi-major axis of Io: 421 800 km

$$\delta = +0.115\%$$

Independent Io estimates obtained through the proton and electron branches.

4. Electron → Mercury

$$\bar{\lambda}_e = 3.8615926744 \times 10^{-13} \text{ m}$$

$$a_{\text{Mercury}} = 0.38709893 \text{ AU} = 5.7909175678 \times 10^{10} \text{ m}$$

$$\Lambda_e = \frac{a_{\text{Mercury}}}{\bar{\lambda}_e} = \frac{5.7909175678 \cdot 10^{10}}{3.8615926744 \cdot 10^{-13}} = 1.49962 \cdot 10^{23}$$

$$\text{Verification: } \frac{\Lambda_e \cdot 10^{-12}}{1 \text{ AU}} = \frac{1.49962 \cdot 10^{11}}{1.495978707 \cdot 10^{11}} = 1.00243 \quad (\delta = +0.243\%)$$

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$$\Delta_{\text{hadron}} = a_{\text{Mercury}} - \bar{\lambda}_e^{\text{norm}} = 0.387099 - 0.386159 = 0.000939 \quad (\equiv m_n \cdot 10^{-6} \text{ MeV})$$

5. Mercury → Io → Europa → Ganymede → Callisto

$$\begin{pmatrix} a_{\text{Io}} \\ a_{\text{Europa}} \\ a_{\text{Ganymede}} \\ a_{\text{Callisto}} \end{pmatrix} = \frac{a_{\text{Mercury}}}{\alpha^{-1}} \cdot \begin{pmatrix} 1 \\ \sqrt{\chi} \\ \chi \\ \phi^3 + \dfrac{31}{137} \end{pmatrix}$$

$$\frac{a_{\text{Mercury}}}{\alpha^{-1}} = \frac{57\,909\,176}{137.036} \approx 422\,583 \text{ km}$$

Table 1: Macroscopic Projection

Satellite	UKF (km)	JPL/NASA (km)	Deviation
Io	422 583	421 800	+0.19%
Europa	672 292	671 100	+0.18%
Ganymede	1 069 557	1 070 400	−0.08%
Callisto	1 885 711	1 882 700	+0.16%

6. Io – Pan – Daphnis – Atlas – Titan

$$\begin{pmatrix} a_{\text{Pan}} \\ a_{\text{Daphnis}} \\ a_{\text{Atlas}} \end{pmatrix} \equiv a_{\text{inner Saturn}} = a_{\text{Io}} \cdot \frac{M_5}{M_7 - M_5} = a_{\text{Io}} \cdot \frac{31}{96} \approx 136\,458 \text{ km}$$

Table 2: Saturnian Cascade

Satellite	**UKF (km)**	**JPL/NASA (km)**	**Deviation**
Pan	136 458	133 584	+2.15%
Daphnis	136 458	136 505	−0.03%
Atlas	136 458	137 670	−0.88%
Mean	**136 458**	**135 920**	**+0.40%**

$$a_{\text{Io}} = a_{\text{Titan}} \times \frac{\sqrt{3}}{5} = 1\,221\,870 \text{ km} \times 0.346410 = 423\,268.18 \text{ km} \quad (\delta = +0.37\%)$$

Uranus Reverse Projection

$$a_{\text{ring}} = a_{\text{Titania}} \times \frac{31^2}{2^{13} - 1} = 435\,910 \times \frac{961}{8191} = 51\,143 \text{ km}$$

Observed ε-ring core radius: 51 149 km. Deviation: -0.012%.

The same register ratio that appears in the inner cascades also recovers the principal ring scale of Uranus.

Venus: Inner-System Inversion

$$a_{\text{Venus}} = 1 \text{ AU} \times \frac{\varphi}{\sqrt{5}} = 0.723607 \text{ AU}$$

Observed semi-major axis of Venus (NASA): 0.723332 AU. Deviation: $+0.038\%$.

The same register ratio that appears in the inner cascades also recovers the orbit of Venus.

Golden Bridge (Photospheric Projection)

$$B_\phi = \varphi + \frac{31}{1000} = 1.649034$$

$$R_\odot^{\text{pred}} = R_{\text{core}} \times B_\phi = 421\,560 \times 1.649034 = 695\,167 \text{ km}$$

Empirical bridge coefficient 31/1000

Modern helioseismic and transit measurements determine the actual physical radius of the solar photosphere as:

$$R_\odot \approx 695\,700 \text{ km}$$

The relative deviation is:

$$\delta = \frac{695\,167 - 695\,700}{695\,700} \approx -0.076\%$$

Deviation: -0.076%. The operator links the proton-derived core scale to the visible photosphere.

System	Operator	UKF Prediction	Deviation
Uranus ϵ-ring	$a_{\text{Titania}} \times \dfrac{31^2}{2^{13}-1}$	51 143 km	-0.012%
Venus orbit	$1\,\text{AU} \times \dfrac{\phi}{\sqrt{5}}$	0.723607 AU	$+0.038\%$
Solar photosphere	$R_{\text{core}} \times \left(\phi + \dfrac{31}{1000}\right)$	695 167 km	-0.076%

Electron–Mercury Correspondence and the Origin of the Astronomical Unit

Within the UKF address hierarchy the Sun is assigned the role of the central proton-scale object and Mercury the role of the corresponding electron-scale object. The numerical content of this assignment is expressed by the ratio of Mercury's semi-major axis to the reduced Compton wavelength of the electron:

$$\bar{\lambda}_e = 3.8615926744 \times 10^{-13}\,\text{m},$$
$$a_{\text{Mercury}} = 5.7909175678 \times 10^{10}\,\text{m}.$$

Their quotient defines the pure dimensionless address

$$\Lambda_e = \frac{a_{\text{Mercury}}}{\bar{\lambda}_e} = 1.49962 \times 10^{23}. \tag{1}$$

Rescaling by twelve orders of magnitude recovers the astronomical unit:

$$\frac{\Lambda_e \times 10^{-12}}{1\,\text{AU}} = 1.00243 \quad (\delta = +0.243\,\%). \tag{2}$$

The electron–Mercury correspondence is *not* introduced as a derivation of Mercury's orbit from the electron Compton wavelength. Rather, the observed Mercury semi-major axis defines a dimensionless address when expressed in reduced electron-Compton units. This address then exhibits a numerical correspondence with the astronomical unit under the framework's 10^{-12} rescaling:

$$\bar{\lambda}_e \;\longrightarrow\; \Lambda_e = \frac{a_{\text{Mercury}}}{\bar{\lambda}_e} \;\longrightarrow\; \Lambda_e \times 10^{-12} \approx 1\,\text{AU}. \tag{3}$$

Once the Mercury address is fixed, the same register operators (α^{-1}, χ, Mersenne ratios) generate the Galilean satellite hierarchy. Two independent geometric routes yield the Io address:

- **Proton branch:**

$$a_{\text{Io}} = \lambda_p \times \Lambda_{\text{UKF}} = 421\,314\,\text{km} \quad (\delta = -0.115\,\%).$$

- **Electron branch:**

$$a_{\text{Io}} \approx \frac{a_{\text{Mercury}}}{\alpha^{-1}} \approx 422\,583\,\text{km} \quad (\delta = +0.19\,\%).$$

The two routes, starting from the proton Compton wavelength and from the electron Compton wavelength respectively, converge on the same physical orbit to within $\sim 0.3\,\%$.

The construction therefore supplies an explicit numerical bridge between the electron Compton scale, the Mercury address, the astronomical unit and the Io address — all remaining geometric and employing only the dimensionless address together with the integer register set of the framework.

Related Works

This note belongs to the Universal Kochenov Framework (UKF) series:

1. Quantum Shell Hierarchy of the Inner Solar System: From Mercury ($n = 1$) to Mars ($n = 4$). 10.5281/zenodo.22707601
2. Fractal Cascade: From Proton to Solar Photosphere. 10.5281/zenodo.21970415
3. A Mass-Free Kinematic Topology for Orbital Mechanics. 10.5281/zenodo.21970434
4. A Geometric Rewriting of the Bohr Scale and the $1/n^2$ Spectral Skeleton. 10.5281/zenodo.21970457
5. A Kinematic-Geometric Description of Orbital Dynamics, Relativistic Perihelion Precession, and Solar Light Deflection. 10.5281/zenodo.21970258
6. Orbital Calculator (HTML tool). 10.5281/zenodo.22009713

All papers and tools are available as preprints on Zenodo under the UKF series.