A Mass Free Kinematic Topology for Orbital Mechanics

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Abstract

This document presents a force-free, geometric framework for orbital mechanics. Masses and the gravitational constant G do not appear in the working equations. Instead, the framework uses dimensionless geometric addresses (a), central-body invariants (R_{eq}, v_0, J_2), and density-based barycentric corrections. The algorithm accurately handles isolated and binary systems; a small $O(10^{-4})$ cascade term is available for three-body configurations. The Earth–Moon–Sun hierarchy uses a dedicated S-E-M operator with a separate Earth calibration ($T_0 = 0.058913$ d) to recover the sidereal month to +0.045%.

1 Core Paradigm & Postulates

The UKF (Unified Kochenov Framework) algorithm bypasses classical force equations, mass parameters (M), and the gravitational constant (G). Orbital systems are modeled as pure kinematic address hierarchies dictated by the physical dimensions and structural densities of the bodies involved. All spatial lengths are converted into a dimensionless coordinate system scaled directly to the equatorial profile of the central parent body.

2 Standard Reference Constants

The algorithm selects system parameters based strictly on the designated Parent attribute.

Table 1: Parent Body Constants

Parent	$\mathbf{R}_{\text{eq}}$ (km)	$\mathbf{v_0}$ (km/s)	$\mathbf{T_0}$ (days)	$\mathbf{J_2}$	ρ (g/cm^3)
Sun (via Earth)	696340.0 (cascade only)	29.7847	365.256	0.0	1.408
Earth	6378.137	7.90537	0.0586728	0.00108263	5.515
Mars	3396.200	3.55115	0.069549	0.0019604	3.934
Jupiter	71492.000	42.1000	0.123500	0.0147360	1.326
Saturn	60268.000	25.1000	0.174800	0.0162980	0.687
Uranus	25559.000	15.0600	0.123420	0.0033412	1.270
Neptune	24764.000	16.6200	0.108360	0.0034000	1.638
Pluto	1188.300	0.85600	0.100950	0.0000100	1.854

Note: R_{eq} for the Sun is used strictly as a structural physical radius within Step C.2 (Cascade calculations). For Step A (Address construction), the Solar field continues to implicitly scale via Earth parameters (1 AU).

3 Step-by-Step Calculation Procedure

3.1 Step A: Dimensionless Address Construction

Map the raw semi-major axis (a_{raw}) into a scale-free spatial address (a).

$$a = \begin{cases} a_{\text{raw}}, & \text{Parent} = \text{Sun} \\ \dfrac{a_{\text{raw}}}{R_{\text{eq}}}, & \text{Parent} = \text{Planet} \end{cases} \tag{1}$$

3.2 Step B: Base Period Approximation

Determine the fundamental unperturbed Keplerian cycle time:

$$T_{\text{base}} = T_0 \cdot a^{1.5} \tag{2}$$

Note: For planet parents, $T_0 = 2\pi R_{\text{eq}}/(86400 v_0)$.

3.3 Step C: Generalized Barycentric & Cascade Corrections

Evaluate multi-body geometric field distortions through the unified scaling index μ:

Binary Pair Mode (e.g., Pluto–Charon, Earth–Moon):

$$\mu = 1 + \left(\frac{R_{\text{patient}}}{R_{\text{eq}}}\right)^3 \cdot \frac{\rho_{\text{patient}}}{\rho_{\text{parent}}} \tag{3}$$

Three-Body Cascade Mode (e.g., Sun–Jupiter–Saturn, Sun–Earth–Moon):

$$\mu = 1 + \left(\frac{R_{\text{perturber}}}{R_{\text{parent}}}\right)^3 \cdot \frac{\rho_{\text{perturber}}}{\rho_{\text{parent}}} \cdot \left(\frac{a_{\text{perturber}}}{a_{\text{target}}}\right)^2 \tag{4}$$

Isolated Small Satellite Mode: Set $\mu = 1.0$.

Compute the intermediate barycentric period:

$$T_{\text{intermediate}} = \frac{T_{\text{base}}}{\sqrt{\mu}} \tag{5}$$

Sensitivity note. The cascade term yields $\mu - 1 \approx 3.0 \times 10^{-4}$ and changes the period by $\sim 0.015\%$. The residual relative to the observed sidereal period ($\sim 10759\,\text{d}$) is dominated by the adopted semi-major axis, not by the cascade factor: matrix $a = 9.53707$ gives $T_{\text{final}} = 10756.1\,\text{d}$ ($\delta \approx -0.03\%$); Standish DE200 $a = 9.54150$ gives $T_{\text{final}} \approx 10763.6\,\text{d}$ ($\delta \approx +0.04\%$). The correction is dimensionless and uses only radii, densities, and addresses.

3.4 Step D: Oblateness (J) Quadrupole Filter

For close-proximity addresses ($a < 5$), compensate for equatorial bulge geometry:

$$T_{J_2} = \frac{T_{\text{intermediate}}}{\sqrt{1 + \frac{3}{2} J_2 \left(\frac{1}{a}\right)^2}} \tag{6}$$

3.5 Step E: Spin-Vector Alignment Optimization

Apply the axial orientation filter. For retrograde orbits only, use:

$$\eta_{\text{spin}} = 1 + J_2 \cdot \frac{1}{a} \cdot \Phi_{\text{system}} \tag{7}$$

$$T_{\text{final}} = \frac{T_{J_2}}{\eta_{\text{spin}}} \tag{8}$$

Note: $\Phi_{\text{system}} = -1$ for retrograde orbits (e.g., Triton). For all prograde and isolated satellites, $\eta_{\text{spin}} = 1$ is applied by default; the formula above is used exclusively for retrograde objects.

3.6 Step F: Kinematic Velocity Profiles

Calculate the orbital velocity matrix over the geometric address domain:

$$v_{\text{mean}} = \frac{v_0}{\sqrt{a}} \left(1 - \frac{e^2}{4} - \frac{3e^4}{64}\right) \tag{9}$$

$$v(\theta) = v_0 \sqrt{\frac{2}{r(\theta)} - \frac{1}{a}}, \quad r(\theta) = \frac{a(1-e^2)}{1 + e\cos\theta} \tag{10}$$

3.7 Step G: Special Operator S-E-M (Earth–Moon–Sun)

The generic three-body cascade of Eq. (4) is unstable for the Earth–Moon–Sun hierarchy because of the extreme radius ratio $R_{\text{Sun}}/R_{\text{Earth}} \approx 109$. UKF therefore defines a dedicated geometric operator:

$$\mu_{\text{SEM}} = \mu_{\text{binary}} \times \mu_{\text{solar}} \tag{11}$$

where

$$\mu_{\text{binary}} = 1 + \left(\frac{R_{\text{Moon}}}{R_{\text{Earth}}}\right)^3 \frac{\rho_{\text{Moon}}}{\rho_{\text{Earth}}} \tag{12}$$

$$\mu_{\text{solar}} = 1 + \left(\frac{R_{\text{Sun}}}{r_{\text{Earth-Sun}}}\right) \cos^2 i \tag{13}$$

with $i = 5.14°$ (inclination of the lunar orbit to the ecliptic), and $r_{\text{Earth-Sun}} = 149597870.7$ km. The operator is activated when $R_{\text{pert}}/R_{\text{parent}} > 50$ and does not replace the universal cascade used for all other systems.

Numerical execution (fixed constants derived from JPL DE200):

Table 2: S-E-M Numerical Execution

Step	**Value**
a	60.26838
T_{base}	27.56420 d
μ_{binary}	1.01225
T_{binary}	27.39703 d
μ_{solar}	1.004617
T_{final}	27.33400 d
NASA sidereal	27.32166 d
Residual	+0.045%

Note on the Earth constant. The strict geometric definition $T_0 = 2\pi R_{\text{eq}}/(86400\, v_0) = 0.0586728\,\text{d}$ yields a residual of -0.36%. The value $T_0 = 0.058913\,\text{d}$ used in the S-E-M numerical example is a hierarchical calibration that recovers the observed sidereal month to $+0.045\%$. This difference quantifies the additional geometric content that the simple equatorial address does not capture for the highest volume-ratio satellite in the Solar System. The S-E-M operator is therefore retained as a dedicated hierarchical correction.

4 Empirical Validation Matrix

Table 3: Empirical Validation Summary

System	**Parameter**	**UKF**	$\boldsymbol{\delta}$
Triton (Neptune)	Period	5.8766 d	−0.005%
Saturn (Sun–Jupiter–Saturn)	Period	10763.6 d*	+0.04%
Phobos (Mars)	Period	0.3187 d	−0.05%
Io (Jupiter)	Period	1.7699 d	+0.041%
Europa (Jupiter)	Period	3.5519 d	+0.021%
Titan (Saturn)	Period	15.9569 d	+0.072%
Enceladus (Saturn)	Period	1.3710 d	+0.059%
Titania (Uranus)	Period	8.7046 d	−0.016%
Charon (Pluto)	Period	6.3833 d	−0.061%

*Using Standish DE200 calibration; error $< 0.04\%$ with PDF address.

Note on Charon. Charon is computed in binary mode (Eq. 3) with the true JPL semi-major axis $a_{\text{raw}} = 19596$ km (PLU060 catalog). The result is $T_{\text{UKF}} = 6.3833$ d, $\delta = -0.061\%$, which is consistent with the binary geometry of the Pluto–Charon system.

5 Conclusion

UKF provides a force-free, geometric framework for orbital mechanics. The algorithm accurately handles isolated and binary systems; the cascade term and the S-E-M operator cover three-body configurations. Masses and G do not appear in the working equations.

Related Works

This note belongs to the Universal Kochenov Framework (UKF) series:

1. Quantum Shell Hierarchy of the Inner Solar System: From Mercury ($n = 1$) to Mars ($n = 4$). 10.5281/zenodo.22707601
2. Fractal Cascade: From Proton to Solar Photosphere. 10.5281/zenodo.21970415
3. A Mass-Free Kinematic Topology for Orbital Mechanics. 10.5281/zenodo.21970434
4. A Geometric Rewriting of the Bohr Scale and the $1/n^2$ Spectral Skeleton. 10.5281/zenodo.21970457
5. A Kinematic-Geometric Description of Orbital Dynamics, Relativistic Perihelion Precession, and Solar Light Deflection. 10.5281/zenodo.21970258
6. Orbital Calculator (HTML tool). 10.5281/zenodo.22009713

All papers and tools are available as preprints on Zenodo under the UKF series.