Hydrogen Level Addresses in the UKF Kinematic Framework: A Geometric Rewriting of the Bohr Scale and the $1/n^2$ Spectral Skeleton

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Abstract

The Universal Kochenov Framework (UKF) represents Bohr-level radii as dimensionless spatial addresses relative to the proton rms charge radius. The address sequence follows

$$a_n = a_1 n^2,$$

while the associated kinematic velocity profile is

$$v_n = \frac{v_1}{n}.$$

Consequently, the squared velocities form an exact $1/n^2$ sequence, and the positive squared-velocity difference for an emission transition $n_i \to n_f$, with $n_i > n_f$, is

$$v_f^2 - v_i^2 = v_1^2 \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right).$$

This reproduces the rational structure of the Rydberg spectral difference in purely kinematic variables.

A quantitative comparison with standard vacuum Ritz wavelengths shows that a single reduced-mass correction shifts the residuals of the tested hydrogen and deuterium transitions from approximately -531 ppm to the interval $+5.4 \ldots + 13.2$ ppm. A fixed-form empirical correction of order $\alpha^2 Z^2/n_f$, applied without line-specific fitting, further reduces the residuals of the tested H, D, and Li III transitions to approximately $-0.9 \ldots + 5.3$ ppm.

The Rydberg constant follows from the same geometric variables:

$$cR_\infty = \frac{\alpha c}{4\pi a_0}.$$

Within the full UKF construction, α is generated from the discrete register $\mathcal{R} = \{31, 144, 720, \phi\}$, while a_1 specifies the normalized ground-state address and the physical scale is set by the parent radius R_p via $a_0 = a_1 R_p$. Thus the spectral skeleton and the absolute frequency scale are both determined by the geometric address structure.

1 Parent Scale and Base Kinematics

In the Universal Kochenov Framework, physical systems are represented through dimensionless spatial addresses relative to a designated parent scale.

For the hydrogen atom, the parent scale is taken as the proton rms charge radius,

$$R_p \approx 0.8412 \times 10^{-15} \text{ m}.$$

The Bohr radius is

$$a_0 \approx 5.29177 \times 10^{-11} \text{ m}.$$

The dimensionless ground-state address is therefore

$$a_1 = \frac{a_0}{R_p} \approx 6.292 \times 10^4.$$

The corresponding base kinematic velocity is defined as

$$v_1 = c\alpha,$$

where c is the speed of light and α is the fine-structure constant. Numerically,

$$v_1 \approx 2187.69 \text{ km s}^{-1}.$$

1.1 Geometric origin of the reference quantities

The quantity α is not introduced here as an independent fitted atomic constant. Within the full UKF construction, the fine-structure ratio is generated from the discrete register

$$\mathcal{R} = \{31,\ 144,\ 720,\ \phi\},$$

where $\phi = (1 + \sqrt{5})/2$, through the independently defined relation:

$$\alpha_{\text{UKF}}^{-1} = 137 + \frac{31^2/144}{(720/120) \cdot 31 - 1/\phi}.$$

Numerically,

$$\alpha_{\text{UKF}}^{-1} = 137.035999246608\ldots$$

Compared with the CODATA 2022 value

$$\alpha^{-1} = 137.035999177(21),$$

the difference is

$$\Delta = 6.96 \times 10^{-8},$$

corresponding to approximately 0.51 ppb.

The normalized ground-state address $a_1 = a_0/R_p$ specifies the geometric scale. The physical Bohr scale is then obtained by the parent-scale map:

$$a_0 = a_1 R_p.$$

Accordingly, the Rydberg frequency can be written as:

$$cR_\infty = \frac{\alpha_{\text{UKF}} c}{4\pi a_1 R_p}.$$

The spectral scale is therefore generated from the UKF register value of α together with the parent-scale realization R_p. The dimensionless architecture determines the spectral ratios and the normalized Bohr address. The parent radius R_p supplies the chosen physical length realization of that address. It is a scale-setting quantity, not a line-specific spectral fit.

No independently fitted spectroscopic constant is introduced as an input.

2 Level Addresses and Radial Scales

The physical Bohr-model radial scale associated with principal quantum number n is

$$A_n = a_0 n^2.$$

The corresponding dimensionless UKF address is

$$a_n = \frac{A_n}{R_p}.$$

Consequently,

$$a_n = a_1 n^2.$$

Multiplication by the proton charge radius recovers the Bohr radial scale:

$$A_n = a_n R_p = a_0 n^2.$$

The distinction between A_n and a_n is maintained throughout the paper:

- A_n is a physical radial scale;
- a_n is a dimensionless address.

Table 1: Hydrogen level addresses and kinematic velocities.

n	Address a_n	Bohr radial scale A_n	$v_n = v_1/n$, km/s
1	6.292×10^4	0.0529 nm	2187.69
2	2.517×10^5	0.2117 nm	1093.85
3	5.663×10^5	0.4763 nm	729.23
4	1.007×10^6	0.8467 nm	546.92
5	1.573×10^6	1.3229 nm	437.54

The listed radial scales are Bohr-model characteristic scales and should not be interpreted as exact quantum-mechanical expectation values for arbitrary orbital angular momentum.

3 Hydrogenic Ions

For a one-electron ion with nuclear charge Z, the Bohr-model scaling becomes

$$A_n(Z) \propto \frac{n^2}{Z},$$

and

$$v_n(Z) \propto \frac{Z}{n}.$$

Thus,

$$A_n(Z) = \frac{a_0 n^2}{Z}$$

and

$$v_n(Z) = \frac{Z v_1}{n}.$$

The corresponding nonrelativistic energy scale follows

$$E_n(Z) \propto \frac{Z^2}{n^2}.$$

For example, in the hydrogenic approximation, the ground-state velocity of He^+ is twice the hydrogen value and its ground-state radial scale is reduced by a factor of two.

The present treatment applies only to one-electron systems or to ions that can be treated as hydrogenic to the required accuracy. In particular,

$$\text{Li III} \equiv \text{Li}^{2+}$$

is treated as a one-electron hydrogenic ion with $Z = 3$.

4 Squared-Velocity Ratios

The UKF velocity profile is

$$v_n = \frac{v_1}{n}.$$

Therefore,

$$v_n^2 = \frac{v_1^2}{n^2}.$$

The ratio of squared velocities between two levels is consequently

$$\frac{v_n^2}{v_m^2} = \frac{m^2}{n^2}.$$

This ratio is an exact rational number within the adopted kinematic scaling.

Table 2: Ratios of squared velocities.

Levels	Ratio	Numerical value
$1 \leftrightarrow 2$	4/1	4.0000
$2 \leftrightarrow 3$	9/4	2.2500
$3 \leftrightarrow 4$	16/9	1.777...
$4 \leftrightarrow 5$	25/16	1.5625

The same structure can be written using the address:

$$a_n = a_1 n^2,$$

so that

$$v_n^2 \propto \frac{1}{a_n} \propto \frac{1}{n^2}.$$

5 Emission Transitions and the Rydberg Difference

For an emission transition

$$n_i \to n_f, \qquad n_i > n_f,$$

the final level has the smaller principal quantum number and therefore the larger kinematic velocity:

$$v_f > v_i.$$

The positive squared-velocity difference is

$$\Delta v_{\text{em}}^2 = v_f^2 - v_i^2 = v_1^2 \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right) > 0.$$

For the Lyman-α transition $2 \to 1$,

$$v_1^2 - v_2^2 = v_1^2 \left(1 - \frac{1}{4} \right) = \frac{3}{4} v_1^2.$$

This is the positive emission-oriented form.

The emitted photon energy is then represented by

$$E_\gamma = \frac{1}{2} m_e \eta_H \left(v_f^2 - v_i^2 \right),$$

where $\eta_H = \mu_{\text{red}}/m_e = m_p/(m_p + m_e)$ is the dimensionless reduced-mass factor.

Thus,

$$E_\gamma \propto \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right).$$

This reproduces the $1/n^2$ combinatorial structure of the Rydberg formula.

6 Base Energy Scale and the Rydberg Constant

With

$$v_1 = c\alpha,$$

the fixed-proton Bohr kinetic energy is

$$E_1^{(0)} = \frac{1}{2} m_e v_1^2 = \frac{1}{2} m_e c^2 \alpha^2.$$

Numerically,

$$E_1^{(0)} \approx 13.60569 \text{ eV}.$$

The corresponding wavenumber scale is

$$R_\infty = \frac{\alpha}{4\pi a_0},$$

or equivalently,

$$cR_\infty = \frac{\alpha c}{4\pi a_0}.$$

Within the full UKF construction, this becomes

$$cR_\infty = \frac{\alpha_{\text{UKF}} c}{4\pi a_1 R_p}.$$

Thus the absolute spectral scale is generated from the UKF register value of α together with the parent-scale realization R_p (via $a_1 = a_0/R_p$). The combinatorial $1/n^2$ skeleton follows from the kinematic address structure alone.

7 Quantitative Spectral Test

The computational chain is

$$a_n \to A_n \to v_n \to E_n \to \Delta E \to \lambda.$$

The uncorrected fixed-proton scale gives for Lyman-α:

$$\lambda_{\text{calc}}^{(0)}(2 \to 1) = 121.50227 \text{ nm}.$$

Relative to the adopted vacuum Ritz reference value

$$\lambda_{\text{ref}}(2 \to 1) = 121.56684 \text{ nm},$$

the residual is approximately

$$-531.1 \text{ ppm}.$$

Table 3: Spectral comparison before and after reduced-mass correction.

Transition	Without η_H, nm	With η_H, nm	Reference, nm	Residual with η_H
$2 \to 1$	121.50227	121.56845	121.56684	+13.2 ppm
$3 \to 2$	656.11228	656.46961	656.46580	+5.8 ppm
$4 \to 3$	1874.60650	1875.62744	1875.61270	+7.9 ppm
$5 \to 3$	1281.46929	1282.16720	1282.16030	+5.4 ppm

Applying the single reduced-mass factor $\eta_H = m_p/(m_p + m_e)$ gives:

A single reduced-mass factor moves the tested hydrogenic wavelengths from a systematic offset of approximately -531 ppm into the interval

$$+5.4 \ldots + 13.2 \text{ ppm}.$$

No line-specific adjustment is introduced.

The remaining residuals are treated here as test residuals rather than evidence for an additional dynamical field.

8 Empirical $\mathcal{O}(\alpha^2)$ Kinematic Filter

To investigate the remaining residual pattern, an additional correction of order α^2 is introduced:

$$\delta = \alpha^2 Z^2 \cdot \frac{1}{4n_f} \cdot \frac{A+Z}{2A},$$

where:

- α is the fine-structure constant;
- Z is the nuclear charge;
- A is the mass number;
- n_f is the final principal quantum number.

For ordinary hydrogen, $A = Z = 1$, so the nuclear factor becomes 1.
For deuterium, $A = 2, Z = 1$, the nuclear factor is $3/4$.
For $^7\mathrm{Li}^{2+}$, $A = 7, Z = 3$, the nuclear factor is $5/7$.
The corrected wavelength is defined as

$$\lambda_{\mathrm{corr}} = \frac{\lambda_{\mathrm{UKF}}}{1+\delta}.$$

No line-specific coefficient is fitted after the formula is fixed.

Within the present eight-line test set, the corrected residuals lie approximately in the interval

$$-0.9 \ldots + 5.3 \text{ ppm}.$$

9 Scope of the Empirical Filter

The correction

$$\delta = \alpha^2 Z^2 \cdot \frac{1}{4n_f} \cdot \frac{A+Z}{2A}$$

is recorded as an empirical kinematic filter.

Within the present test set, the filter is effective for:

Table 4: Residuals after the empirical filter.

System	Transition	Residual with η_H	Residual with δ
Hydrogen	$2 \to 1$	+13.21 ppm	−0.10 ppm
Hydrogen	$3 \to 2$	+5.80 ppm	−0.86 ppm
Hydrogen	$4 \to 3$	+7.86 ppm	+3.43 ppm
Hydrogen	$5 \to 3$	+5.38 ppm	+0.95 ppm
Deuterium	$2 \to 1$	+13.13 ppm	+3.15 ppm
Deuterium	$3 \to 2$	+6.30 ppm	+1.30 ppm
Deuterium	$4 \to 3$	+8.60 ppm	+5.28 ppm
Li III	$3 \to 2$	+42.05 ppm	−0.74 ppm

- hydrogen;
- deuterium;
- the tested Li III transition.

For higher-Z hydrogenic ions, the same expression does not automatically guarantee sub-ppm residuals. Such systems remain outside the present claim.

10 Methodological Boundaries

Two methodological boundaries are imposed.

10.1 Macroscopic quadrupole scaling

A macroscopic J_2 correction in celestial mechanics is not identified with atomic fine structure.

If a macroscopic quadrupole-inspired address correction were formally transplanted into the atomic address hierarchy with $\Delta\theta \propto a^{-1}$, then $a_n \propto n^2$ would imply $\Delta\theta \propto n^{-2}$. This formal scaling does not reproduce the n^{-4} order of the standard fine-structure energy correction and is therefore not identified with atomic fine structure.

10.2 Absolute spectral metrology

The UKF address construction supplies the discrete $1/n^2$ spectral skeleton. The absolute frequency scale is generated from the UKF register value of α together with the parent-scale realization R_p (via $a_1 = a_0/R_p$). The combinatorial $1/n^2$ skeleton follows from the kinematic address structure alone.

11 Central Geometric Result

The central result can be summarized as

$$a_n \propto n^2,$$

$$v_n \propto \frac{1}{n},$$

$$v_n^2 \propto \frac{1}{n^2}.$$

For an emission transition $n_i \to n_f$, with $n_i > n_f$,

$$\Delta v_{\mathrm{em}}^2 = v_f^2 - v_i^2 \propto \frac{1}{n_f^2} - \frac{1}{n_i^2}.$$

Thus, the Rydberg $1/n^2$ difference appears as a difference of squared kinematic velocities. The Rydberg constant follows from the same geometric variables:

$$cR_\infty = \frac{\alpha_{\mathrm{UKF}} c}{4\pi a_1 R_p}.$$

12 Diagnostic Controls

The framework can be tested through the following sequence.

Address test

$$\frac{a_n}{a_1} \stackrel{?}{=} n^2.$$

Velocity test

$$\frac{v_n}{v_1} \stackrel{?}{=} \frac{1}{n}.$$

Energy-scale test

$$\frac{E_n}{E_1} \stackrel{?}{=} \frac{1}{n^2}.$$

Spectral test

$$\lambda_{\mathrm{calc}} \to \lambda_{\mathrm{ref}}.$$

Uniform reduced-mass test

Apply one value of η_H to the complete hydrogen series without line-specific fitting.

Optional empirical-filter test

Apply the same functional form of δ to all selected H, D, and Li III transitions.

After these controls, the residual map

$$\varepsilon_{\mathrm{ppm}}(n_i, n_f, Z, A)$$

can be examined for systematic dependence on the principal quantum numbers, nuclear charge, and mass number.

13 Conclusion

The UKF construction provides a geometric rewriting of the Bohr n^2 address grid and the $1/n^2$ spectral skeleton in kinematic variables.

The principal relations are

$$a_n = a_1 n^2,$$

$$v_n = \frac{v_1}{n},$$

and

$$v_n^2 = \frac{v_1^2}{n^2}.$$

For an emission transition $n_i > n_f$, the positive squared-velocity difference is

$$v_f^2 - v_i^2 = v_1^2 \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right).$$

This reproduces the combinatorial $1/n^2$ structure of the Rydberg formula.

A single reduced-mass correction shifts the tested hydrogen and deuterium residuals from approximately -531 ppm to the low-ppm interval $+5.4 \ldots + 13.2$ ppm. A fixed-form empirical correction of order $\alpha^2 Z^2 / n_f$, applied without line-specific fitting, further compresses the present H, D, and Li III test residuals to approximately $-0.9 \ldots + 5.3$ ppm.

The Rydberg constant follows from the same geometric variables:

$$cR_\infty = \frac{\alpha_{\mathrm{UKF}} c}{4\pi a_1 R_p}.$$

The construction remains a geometric and kinematic rewriting of the Bohr-level structure. The absolute frequency scale is generated from the UKF register value of α together with the parent-scale realization R_p (via $a_1 = a_0/R_p$). The combinatorial $1/n^2$ skeleton follows from the kinematic address structure alone.

Related Works

This note belongs to the Universal Kochenov Framework (UKF) series:

1. Quantum Shell Hierarchy of the Inner Solar System: From Mercury ($n = 1$) to Mars ($n = 4$). 10.5281/zenodo.22707601
2. Fractal Cascade: From Proton to Solar Photosphere. 10.5281/zenodo.21970415
3. A Mass-Free Kinematic Topology for Orbital Mechanics. 10.5281/zenodo.21970434
4. A Geometric Rewriting of the Bohr Scale and the $1/n^2$ Spectral Skeleton. 10.5281/zenodo.21970457
5. A Kinematic-Geometric Description of Orbital Dynamics, Relativistic Perihelion Precession, and Solar Light Deflection. 10.5281/zenodo.21970258
6. Orbital Calculator (HTML tool). 10.5281/zenodo.22009713

All papers and tools are available as preprints on Zenodo under the UKF series.